Excuse Me Sir, Will That Be One Millisecond … Or Two ??

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We are still not sure what possessed Pete to ask such a dumb question last year. Obviously lacking formal geophysical training, he just didn't understand such a simple matter. Fortunately, he had the wisdom to ask the question of some of his more qualified geophysical associates.

So why did a wide spectrum of well-qualified geophysicists provide an even greater diversity of replies? All Pete wanted to know was if he should record his seismic data with sample rates of one millisecond, or two! Perhaps one of his best responses was the one which informed him that 1000 Hertz or 500 Hertz are "sample rates" ... 1 ms or 2 ms are "sample intervals". As correct as that response was, it did not solve Pete's problem.

The replies that Pete received revealed many different perspectives on temporal sampling. Many were valid; others reflected a degree of naivety. All of the replies were enlightening. And the net effect was to confirm that Pete was correct to have asked the question.

There are times when more data is good, and there are times when more is worse. A review of the factors governing selection of sample interval are definitely warranted. Acquisition requirements differ from processing requirements; and those in turn differ from interpretation requirements. At what points is finer sampling required? And at what points is our recovery of information inhibited by the wrong choice?

Nyquist's theorem is well documented, although perhaps not well understood. To sample a sinusoidally oscillating waveform, we must use at least two samples per wavelength (one to describe a positive lobe, and one to describe the negative lobe). Apparently, there are those who believe that these samples must coincide with local "peaks" or "troughs" of a waveform, but this is obviously not the case.

Another way of illustrating Nyquist's theorem is to view the various possible fits of continuous sinusoids to a sparsely sampled data set. Figure 1 shows a 150 Hz sinusoid sampled every two milliseconds. The bottom part of the figure shows the original analogue signal (blue) and its discrete samples (at a 2 ms sample interval). The amplitude of the sinusoid is correctly predicted even though most samples do not coincide with peaks or troughs. The top portion of the diagram shows a spectral analysis of these discrete data. This is produced by cross correlating many mono-frequency sinusoids with the digital data and plotting the correlation coefficient for each frequency. The sidelobes of this spectrogram are due to the fixed time window used in the correlation. Note the strong correlation at 150 Hz. Note also, a similarly strong correlation for a signal of 350 Hz. The middle graph shows the discrete data superimposed on a continuous 350 Hz sinusoid. Both the 150 Hz and the 350 Hz waveforms represent perfect fits. We can expect similarly perfect fits for waveforms of frequencies 650 Hz, 850 Hz, 1150 Hz, 1350, Hz and so on (each even multiple of the Nyquist frequency plus or minus the input frequency).

So presented with these discrete samples, how should we interpret them? Which frequency is the correct one to associate with the digital data? The Nyquist theorem simply indicates that provided we have limited our input analogue data to a bandwidth less than the Nyquist frequency (1 / [2 x Si] where Si is sample interval in seconds), then the lowest frequency alternative of the infinite number of possibilities will be the correct re-construction.

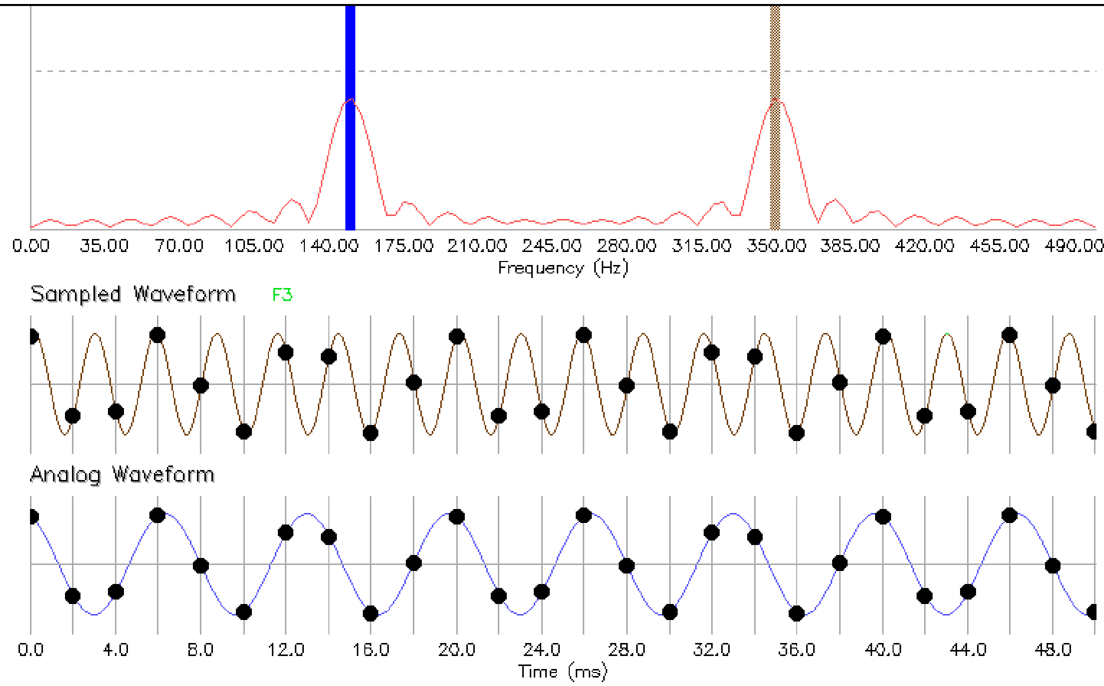


Figure 1
A different view of aliasing.

Discretely sampled sinusoidal data can be reconstructed using many different waveforms. We assume the "correct" reconstruction is the lowest frequency that correlates well with the data. This is a correct assumption provided the input data only contained frequencies lower than the Nyquist frequency.

Seismic recording instrumentation applies "anti-alias" filters to virtually eliminate frequencies above the Nyquist so that no aliased data will distort the frequencies of interest to us. In modern recording instruments, these low pass filters retain data up to 160 to 200 Hz (depending on the make of instrument) without loss in a 2 millisecond sampling mode. Therefore, unless we honestly believe we can recover useful information of higher frequencies than 200 Hz from seismic reflections from our zone of interest, we have no reason to reduce our sample interval (increase our sample rate).

In fact, by reducing our sample interval, we generate more numbers (more bits of information) that must be relayed from box to box in modern distributed telemetry systems. This increased "bit load" can result in transmission failures along the connecting digital cables. This slows acquisition time and increases costs. In some systems, reduced sample intervals can overload the memory (and/or input capacity) of some of the components, requiring additional hardware to be provided (again increasing program costs).

Furthermore, modern "24-bit" recording systems utilize a technology known as Delta-Sigma modulators. These are basically one-bit (sign bit) converters that operate at high clock speeds to produce a stream of bits whose average over time is a very close digital estimate to the analogue input signal. At a two-millisecond sample interval, today's units are 512-times oversampling A-D converters. In other words, they provide an average of 512 measurements that converge to a nearly precise estimate of the analogue input value. At one-millisecond sample intervals, they become only 256-times oversampling. The average output for each sample does not have time to converge as precisely, and we lose 3 dB of our dynamic range. Admittedly, this is a theoretical loss from the least significant bits (which are likely lost amongst noise anyway). However the point remains. Smaller sample intervals are potentially destructive and should not be invoked unless we are reasonably sure that we will recover useable frequencies to justify the action.

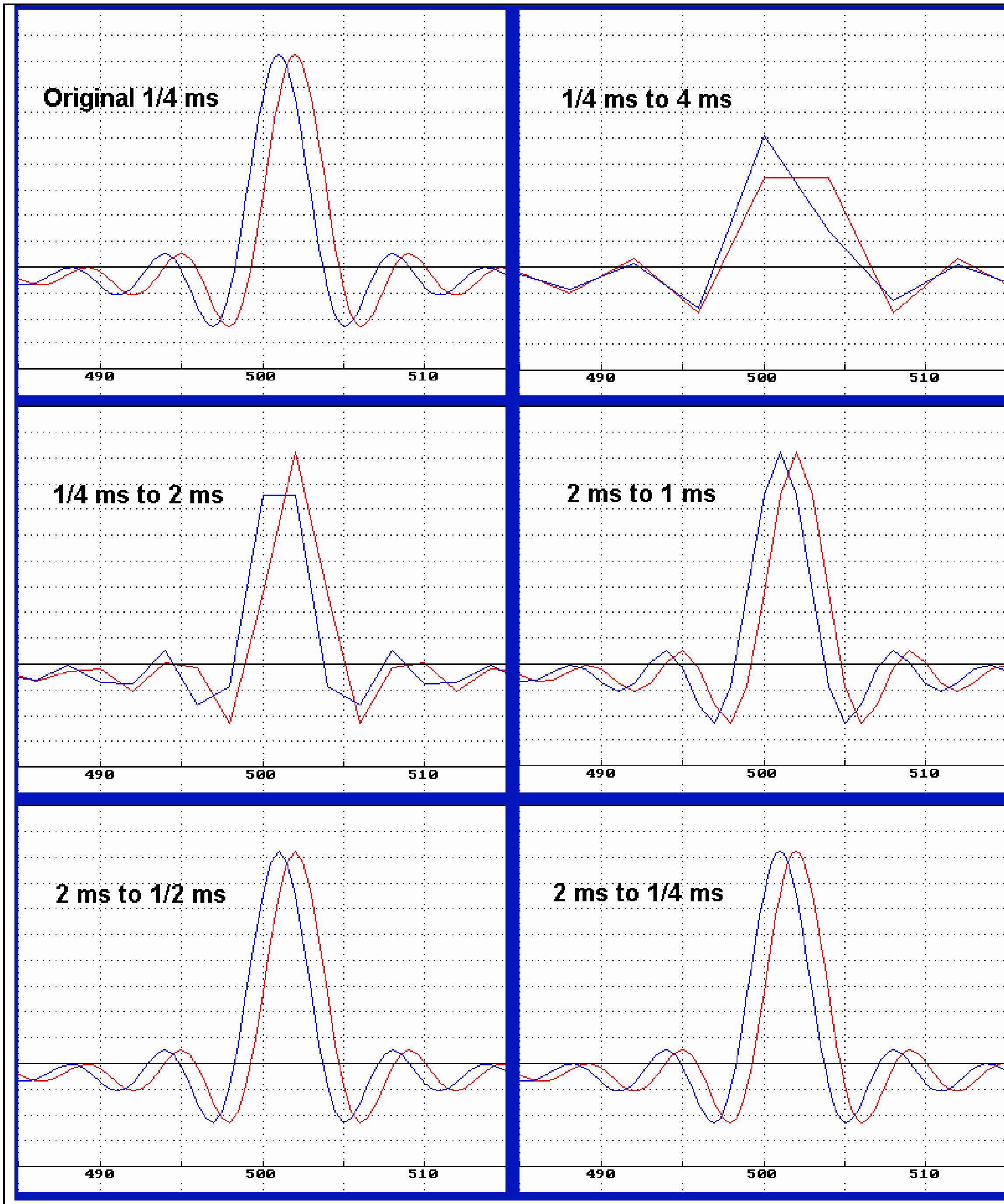


Figure 2

5-15-150-200 Ormsby wavelets centered on 501 and 502 ms respectively.

These digital plots show the loss of character for wavelets with sparse sampling. However, the character can be restored with appropriate resampling.

There are those who claim that wavelet shape and character are lost when large sample intervals are used (see Figure 2). This is largely the result of modern "digital" plotters. If discretely sampled data are plotted with an analogue device, the momentum of the device provides a smooth curve through the discrete points. However, with straight line "connect-the-dots" type of plotting, our wavelets take on a rough appearance. The top left illustration in figure 2 shows two identical wavelets sampled every ¼ ms. The peaks of the two wavelets are separated by one millisecond. However, if we only plot one sample every 4 or 2 milliseconds (top right and middle left, respectively) we can see that the waveforms are distorted and amplitudes appear to change depending on the location of the wavelet. In the extreme case (4 ms) there even appears to be a phase shift.

The remaining illustrations in figure 2 show that the data can be reconstructed with complete accuracy if the 2 ms data is resampled. Recall that basic theory requires down sampling to be accomplished by convolution of the sparse series with a SYNC function (Sin[x] / x) that is sampled at the smaller sample interval. We have seen examples where other curve fitting routines were used (for example cubic spline) and the users have concluded that the resampled product distorted the phase and amplitude compared to the original ¼ ms product. For sinusoidal based data (such as seismic data) we must use a Sin-based interpolator (such as the sync function)!

Unfortunately, many workstations use "connect-the-dots" displays such that when one zooms in to observe wavelet shape, they see unpleasant results at 2 ms sampling. This can easily be corrected by down sampling the data prior to loading on the workstation. It is NOT necessary to reduce the sample interval at the field recording stage.

For processing purposes, the data must be properly sampled according to Nyquist (or at least according to the high cut filter of the recording instruments). The processor cannot legitimately produce data with frequency content in excess of 200 Hz if the data has been recorded at 2 ms with conventional instruments. However, if the final stack is to be limited to 120 Hz, then there is no need to process more than 2 ms sampled data. If you anticipate real useable and recoverable frequencies above the high cut filter of the instruments, then certainly you must choose to reduce the recording sample interval.

We recommend that the recording sample interval be selected such that the highcut filter of the recording system exceeds the highest useful frequency anticipated. The processing sample rate should be selected to maintain a Nyquist frequency higher than the highest frequency anticipated. The display sample rate for workstations should be kept large for most regional work, but data can be resampled to smaller sample intervals for detailed wavelet analysis.

Decreasing sample intervals unnecessarily at the recording or processing stages may result in loss of precision (dynamic range), and will almost certainly result in increased down time in the field, and increased program costs.

Thanks Pete, for asking the question.

Excuse Me Sir …
Will That be
One Millisecond or Two?

Pete MacKenzie (CGAS Inc)

Norm Cooper (Mustagh Resources)

Pete's Question

OK, sorry to bother you folks about something like this, but my pea brain gets confused easily and when I don't understand apparent irrational decisions, I ask.

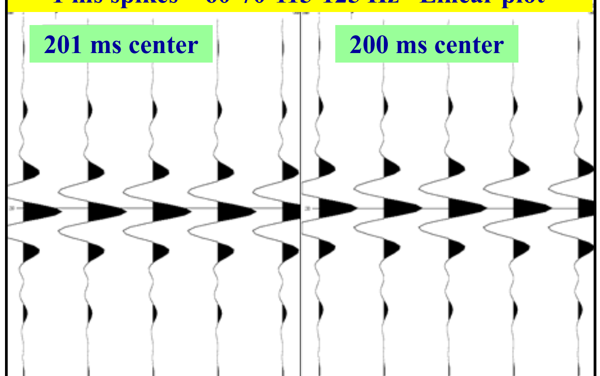
You are all familiar with the Appalachian Basin geophysical response. Can someone explain to me why folks are recording data at one millisecond more and more? Is it defining the curve better? Because the math says it shouldn't (this is geophysics though, and as we all know frequently our model doesn't fit)

Please, I am soliciting your opinions, if you will share.

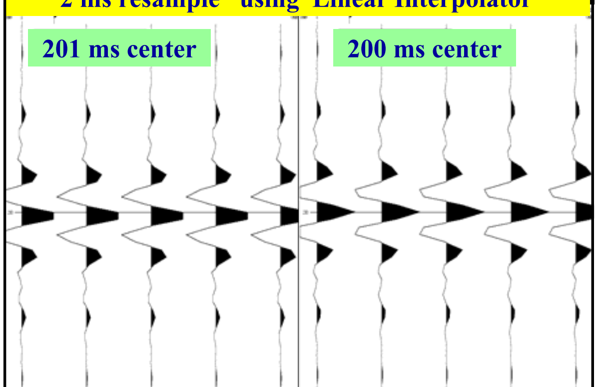
One Responder's Comment

"Mr. Nyquist made the assumption that you would record the wavelet at it's peaks and troughs."

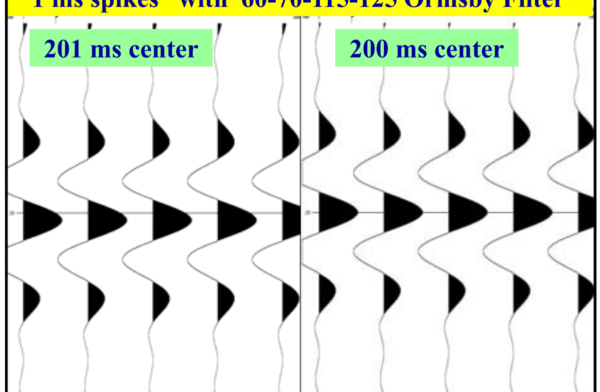
1 ms spikes 60-70-115-125 Hz Linear plot

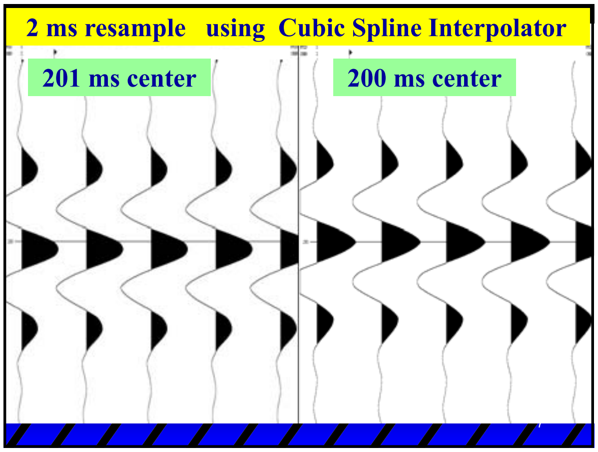


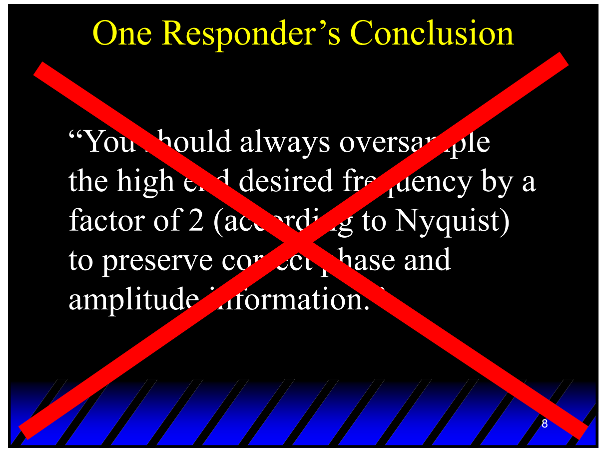
2 ms resample using Linear Interpolator

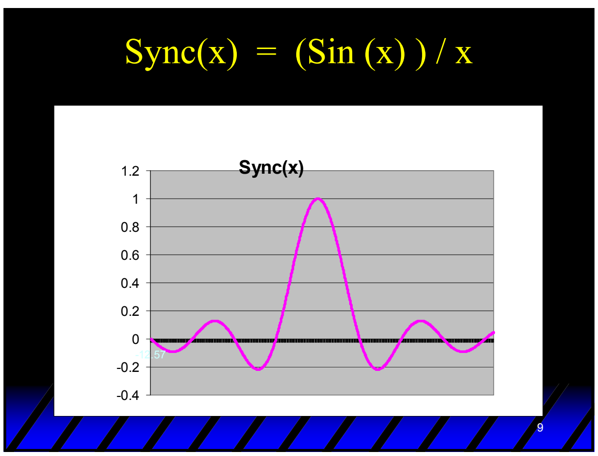


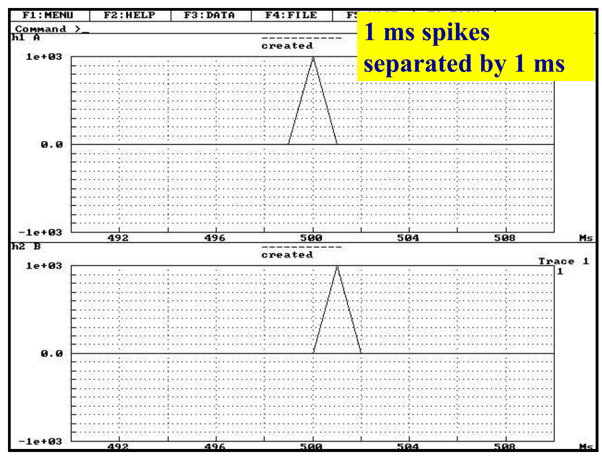
1 ms spikes with 60-70-115-125 Ormsby Filter

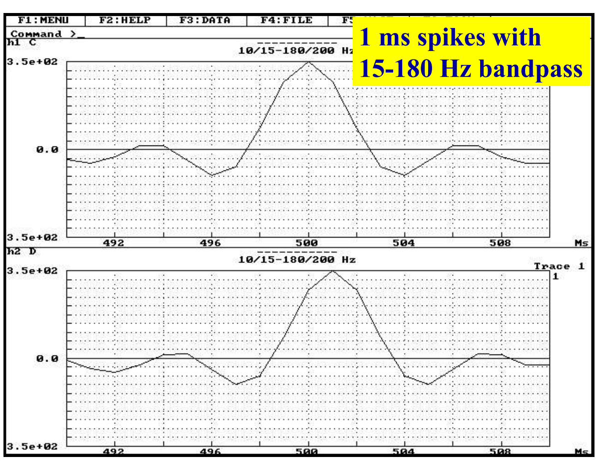


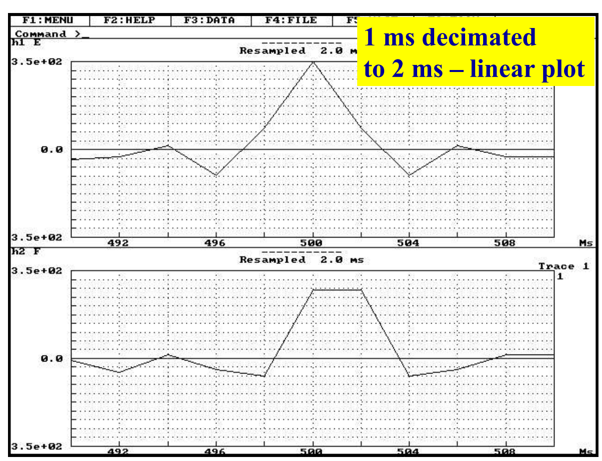

2 ms resample using Cubic Spline Interpolator
201 ms center
200 ms center

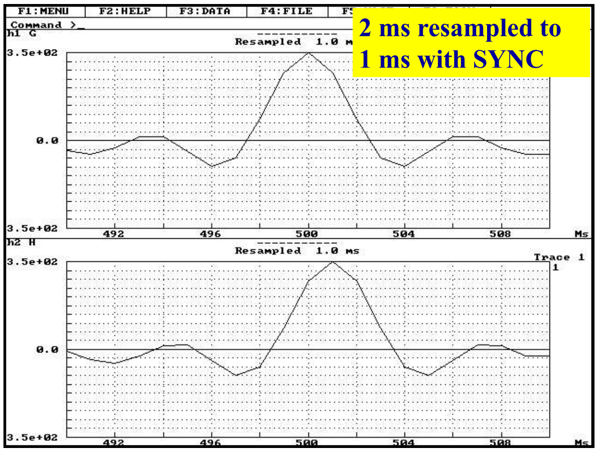

One Responder’s Conclusion
“You hould always oversa ple
the high e d desired fre uency by a
factor of 2 (ac rdi g to Nyquist)
to preserve cor ect hase and
amplitude formation.
8

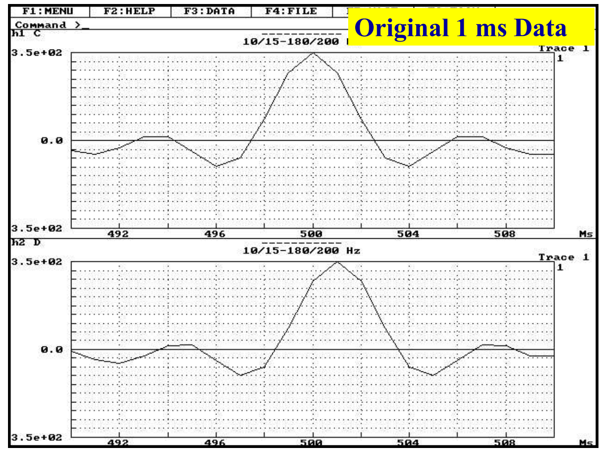

Sync(x) = (Sin (x)) / x
Sync(x)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
9

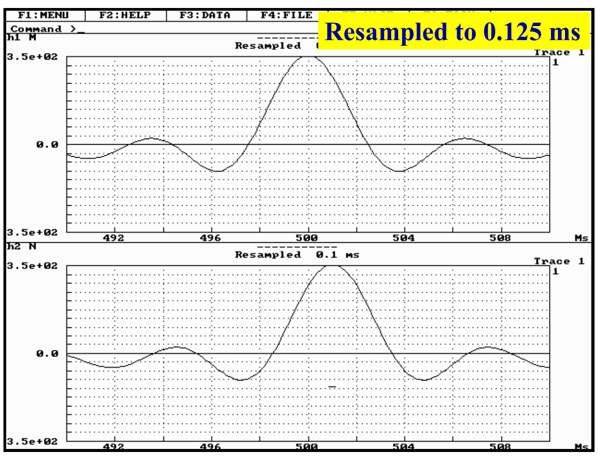

F1:MENU
F2:HELP
F3:DATA
F4:FILE
1 ms spikes
separated by 1 ms
created
1e+03
0.0
-1e+03
492
496
500
504
508
Ms
Trace 1

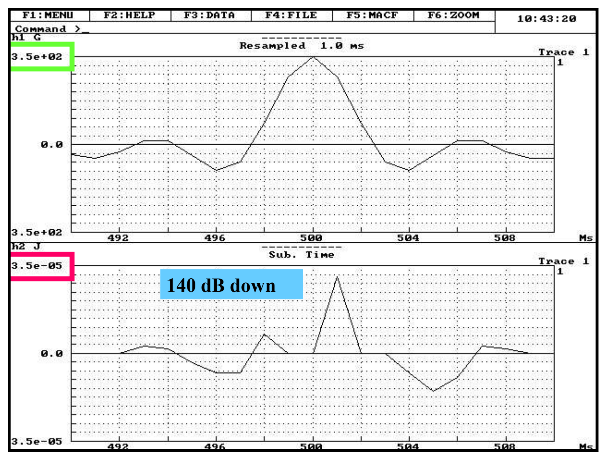

F1:MENU
F2:HELP
F3:DATA
F4:FILE
1 ms spikes with
15-180 Hz bandpass
10/15-180/200 Hz
3.5e+02
0.0
492
496
500
504
508
Ms
Trace 1

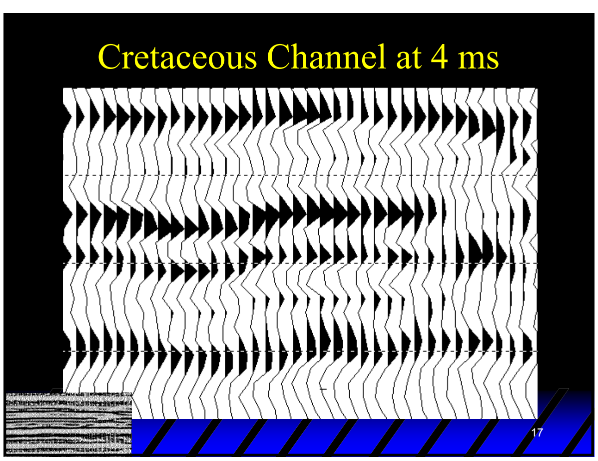

F1:MENU
F2:HELP
F3:DATA
F4:FILE
1 ms decimated
to 2 ms – linear plot
Resampled 2.0 ms
3.5e+02
0.0
492
496
500
504
508
Ms
Trace 1

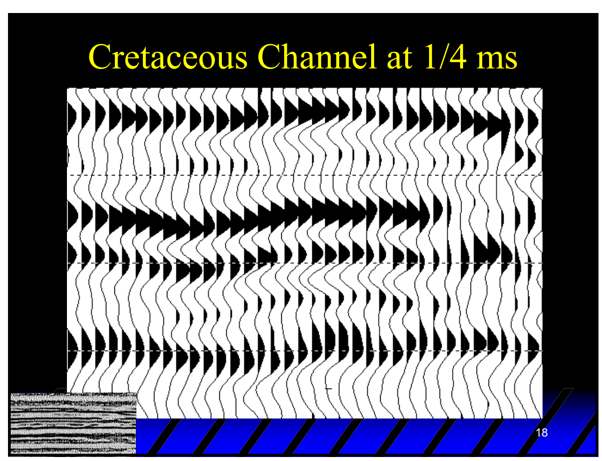

2 ms resampled to
1 ms with SYNC
Resampled 1.0 ms
Trace 1


Original 1 ms Data
10/15-180/200 Hz
Trace 1

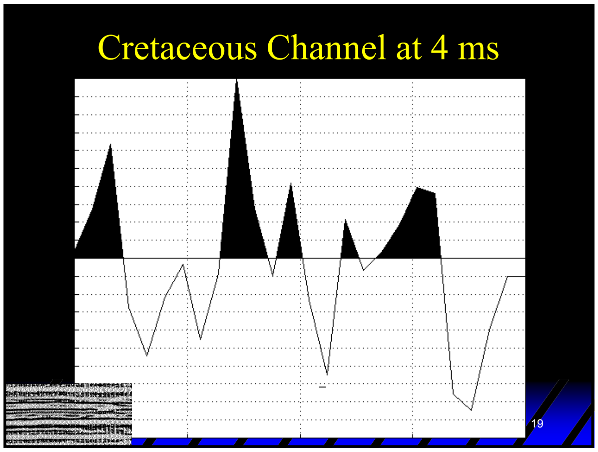

Resampled to 0.125 ms
Resampled 1.0 ms
Resampled 0.1 ms
Trace 1

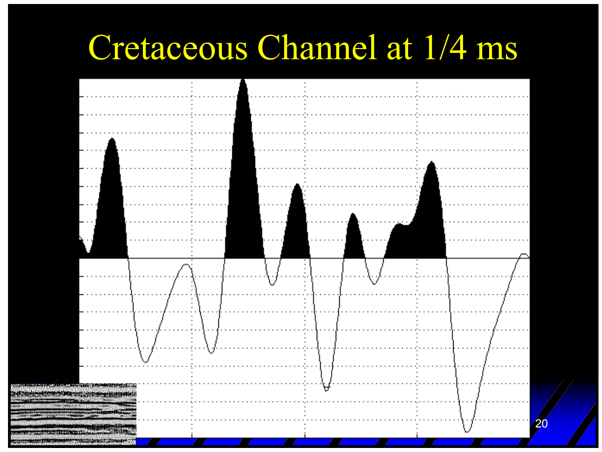

Resampled 1.0 ms
Sub. Time
140 dB down
3.5e+02
3.5e-05
Trace 1

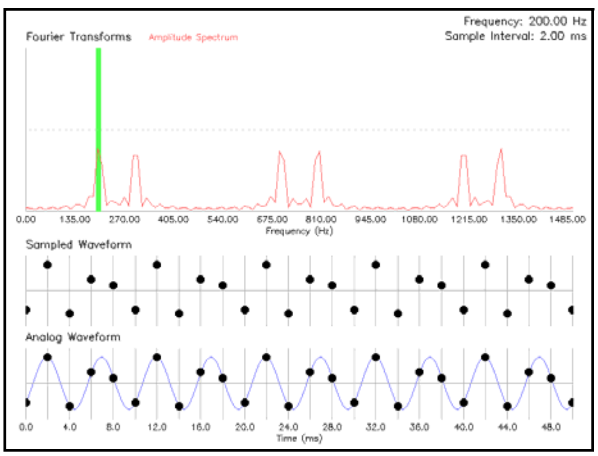

Cretaceous Channel at 4 ms
17

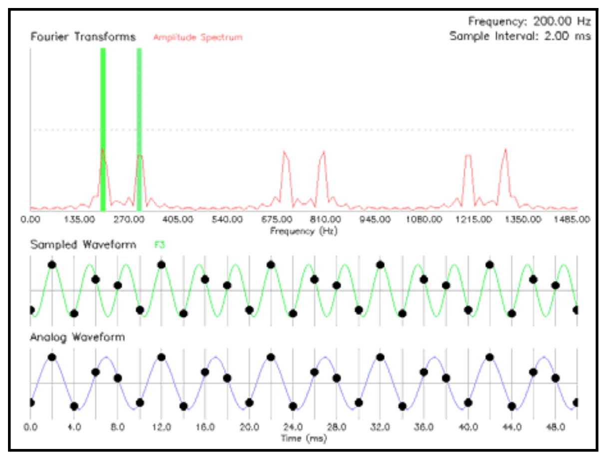

Cretaceous Channel at 1/4 ms
18

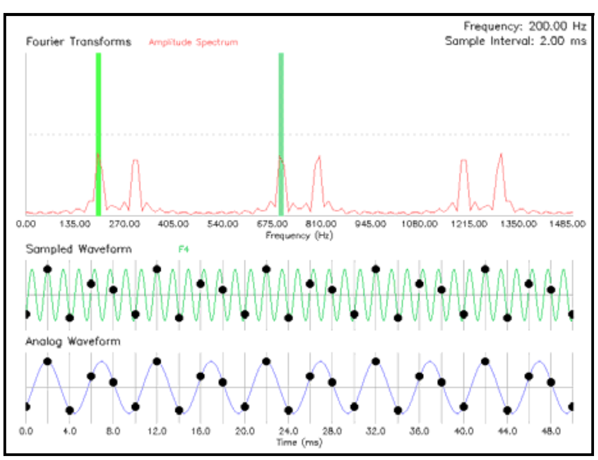
MUSTAGH RESOURCES LTD.

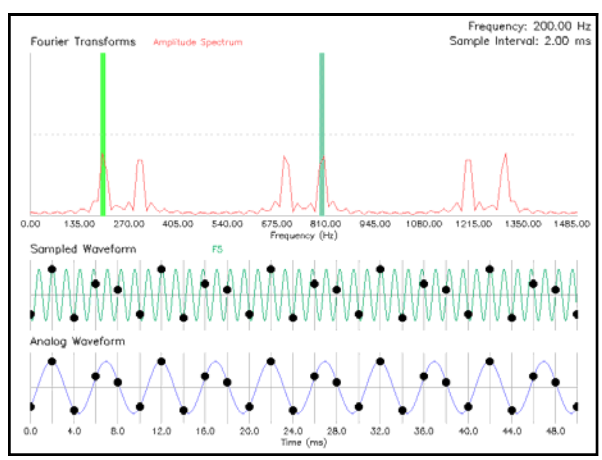


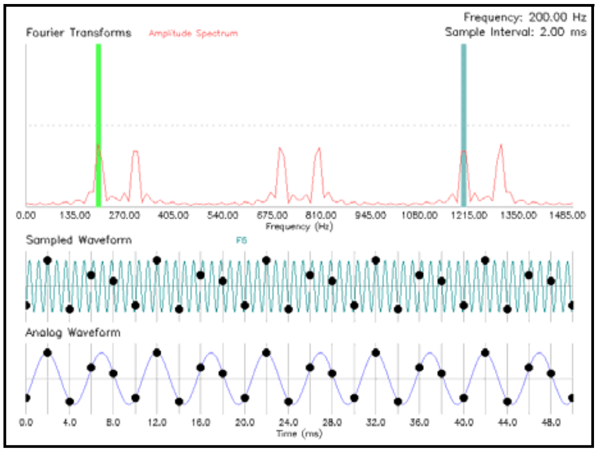


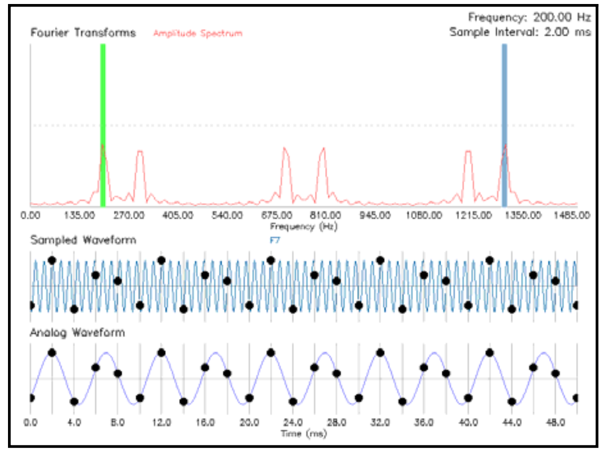


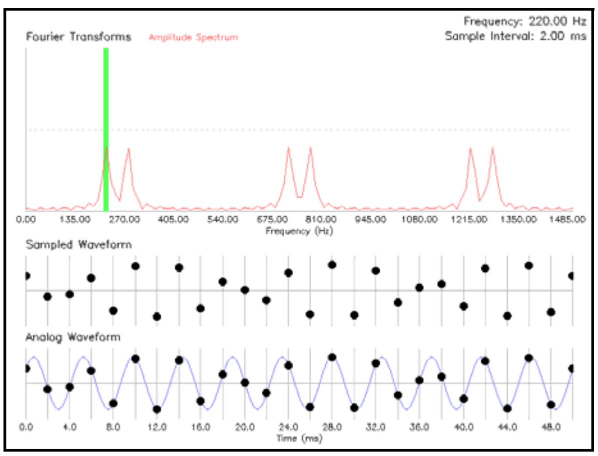


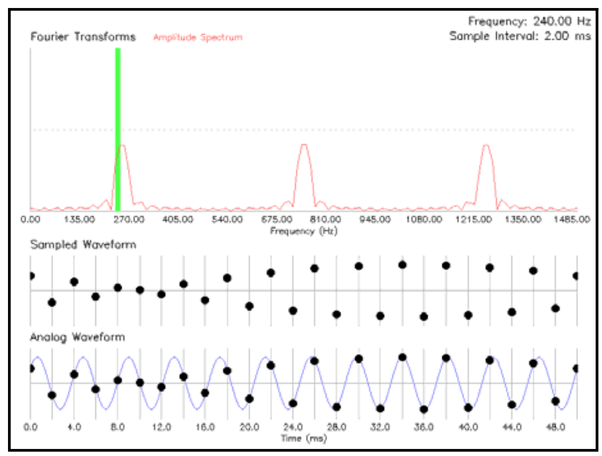


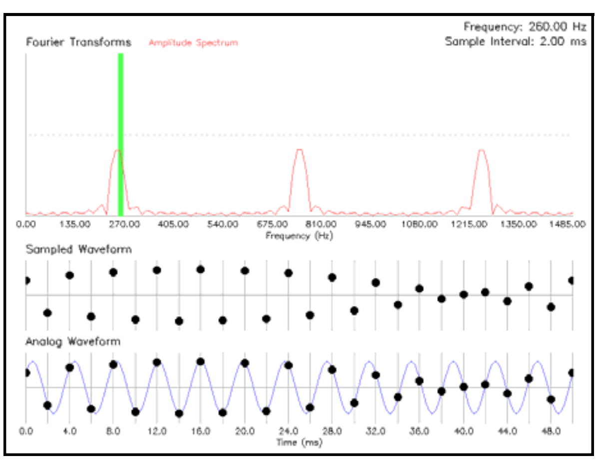


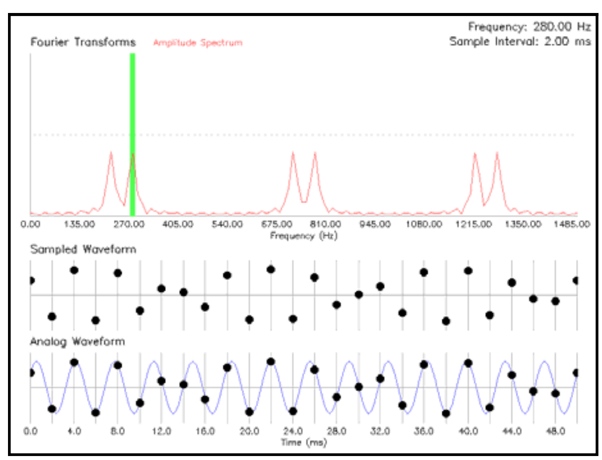

Fourier Transforms
Amplitude Spectrum
Frequency: 200.00 Hz
Sample Interval: 2.00 ms
0.00 135.00 270.00 405.00 540.00 675.00 810.00 945.00 1080.00 1215.00 1350.00 1485.00
Frequency (Hz)
Sampled Waveform
F6
Analog Waveform
0.0 4.0 8.0 12.0 16.0 20.0 24.0 28.0 32.0 36.0 40.0 44.0 48.0
Time (ms)

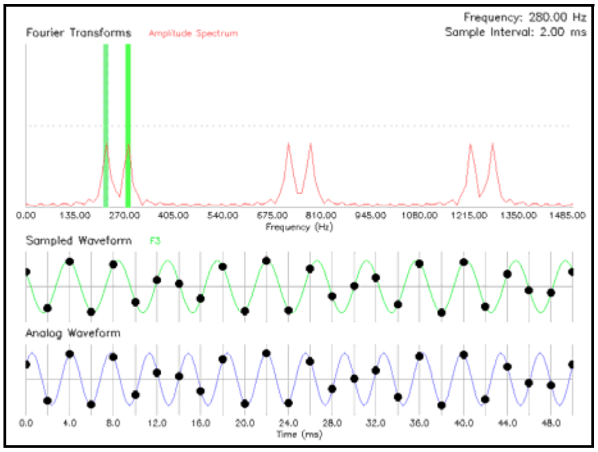

Fourier Transforms
Amplitude Spectrum
Frequency: 200.00 Hz
Sample Interval: 2.00 ms
0.00 135.00 270.00 405.00 540.00 675.00 810.00 945.00 1080.00 1215.00 1350.00 1485.00
Frequency (Hz)
Sampled Waveform
F7
Analog Waveform
0.0 4.0 8.0 12.0 16.0 20.0 24.0 28.0 32.0 36.0 40.0 44.0 48.0
Time (ms)


Fourier Transforms
Amplitude Spectrum
Frequency: 220.00 Hz
Sample Interval: 2.00 ms
0.00 135.00 270.00 405.00 540.00 675.00 810.00 945.00 1080.00 1215.00 1350.00 1485.00
Frequency (Hz)
Sampled Waveform
Analog Waveform
0.0 4.0 8.0 12.0 16.0 20.0 24.0 28.0 32.0 36.0 40.0 44.0 48.0
Time (ms)

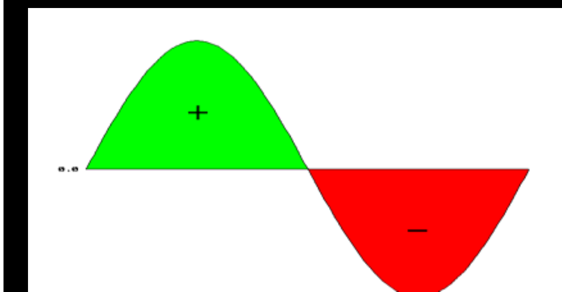

Fourier Transforms
Amplitude Spectrum
Frequency: 240.00 Hz
Sample Interval: 2.00 ms
0.00 135.00 270.00 405.00 540.00 675.00 810.00 945.00 1080.00 1215.00 1350.00 1485.00
Frequency (Hz)
Sampled Waveform
Analog Waveform
0.0 4.0 8.0 12.0 16.0 20.0 24.0 28.0 32.0 36.0 40.0 44.0 48.0
Time (ms)


Fourier Transforms
Amplitude Spectrum
Frequency: 260.00 Hz
Sample Interval: 2.00 ms
0.00 135.00 270.00 405.00 540.00 675.00 810.00 945.00 1080.00 1215.00 1350.00 1485.00
Frequency (Hz)
Sampled Waveform
Analog Waveform
0.0 4.0 8.0 12.0 16.0 20.0 24.0 28.0 32.0 36.0 40.0 44.0 48.0
Time (ms)


Fourier Transforms
Amplitude Spectrum
Frequency: 280.00 Hz
Sample Interval: 2.00 ms
0.00 135.00 270.00 405.00 540.00 675.00 810.00 945.00 1080.00 1215.00 1350.00 1485.00
Frequency (Hz)
Sampled Waveform
Analog Waveform
0.0 4.0 8.0 12.0 16.0 20.0 24.0 28.0 32.0 36.0 40.0 44.0 48.0
Time (ms)



Nyquist Sampling Theory . . .



. . . Requires at least two samples per cycle

33

The Nyquist Theorem

S.I. <= P / 2

P >= 2 x S.I.

F <= 1 / [2 x S.I.]

$F_{Nyquist}$ = 1 / [2 x S.I.]

34

The Nyquist Theorem - Spatially

S.I. <= λ / 2

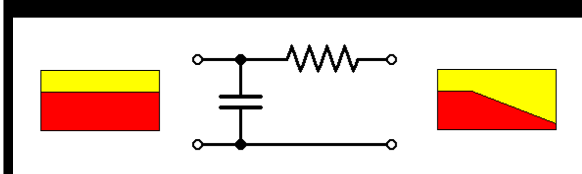
λ >= 2 x S.I.

K <= 1 / [2 x S.I.]

$K_{Nyquist}$ = 1 / [2 x S.I.]

35

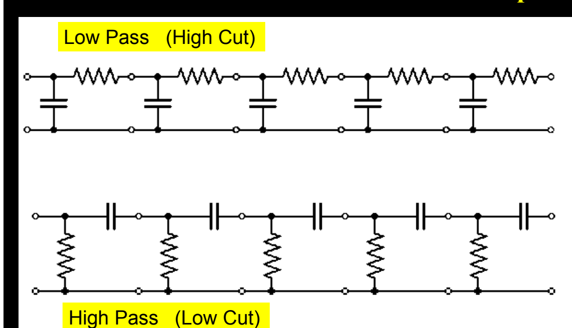
A Basic Analogue Low-Pass Filter



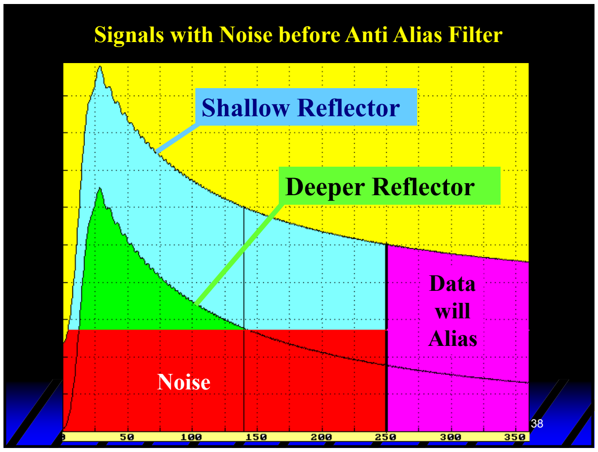
But Cut-Off Slope is only 3 dB / octave

36

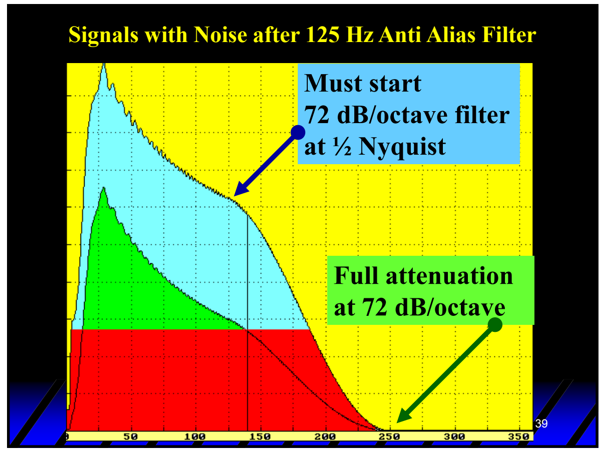
Concatenated Filters Increase Cut-Off Slopes



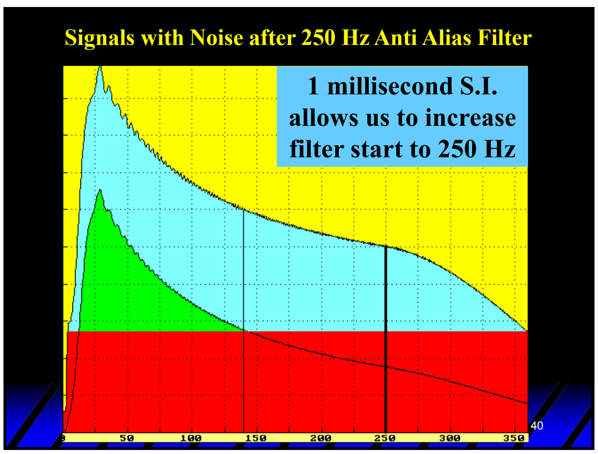
Maximum for stable filter is 72 dB/octave



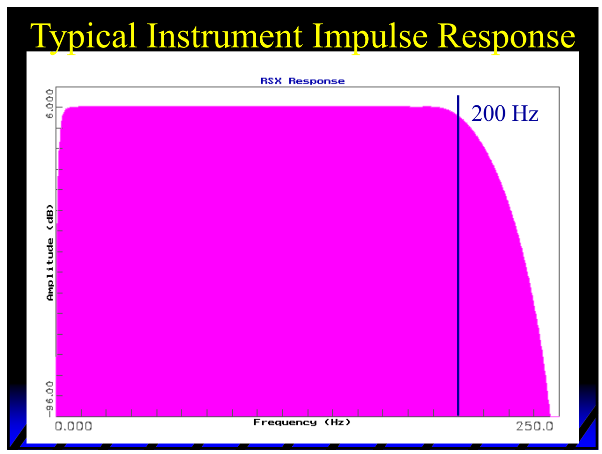
Signals with Noise before Anti Alias Filter
Shallow Reflector
Deeper Reflector
Data
will
Alias
Noise
50
100
150
200
250
300
350
38



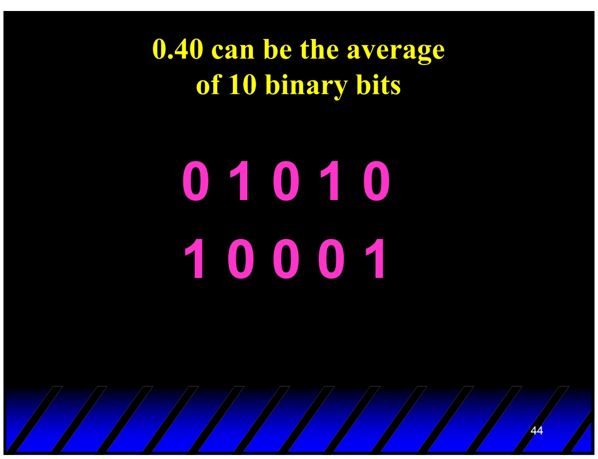
Signals with Noise after 125 Hz Anti Alias Filter
Must start
72 dB/octave filter
at ½ Nyquist
Full attenuation
at 72 dB/octave
50
100
150
200
250
300
350
39



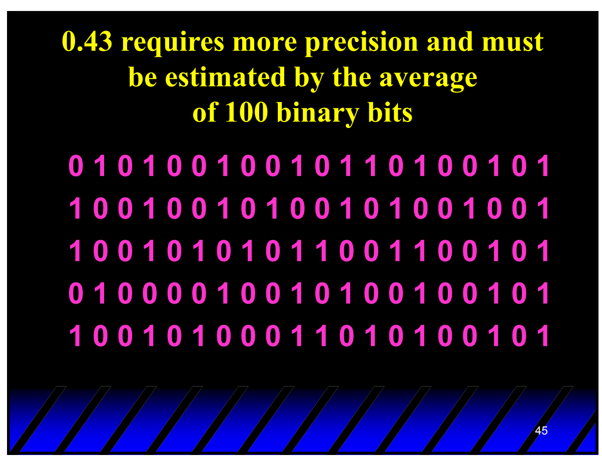
Signals with Noise after 250 Hz Anti Alias Filter
1 millisecond S.I.
allows us to increase
filter start to 250 Hz
50
100
150
200
250
300
350
40



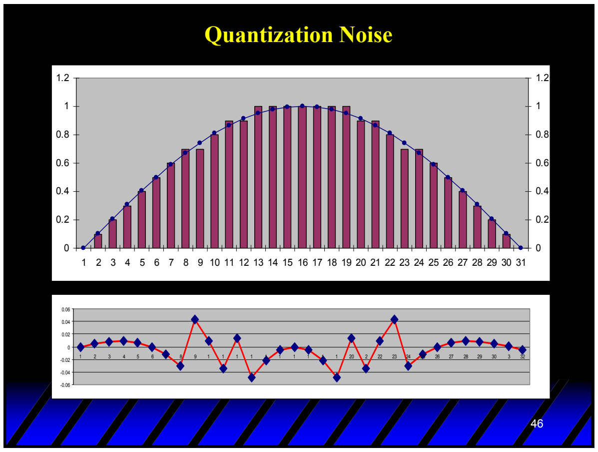
Typical Instrument Impulse Response
RSX Response
200 Hz
Amplitude (dB)
Frequency (Hz)
0.000
250.0

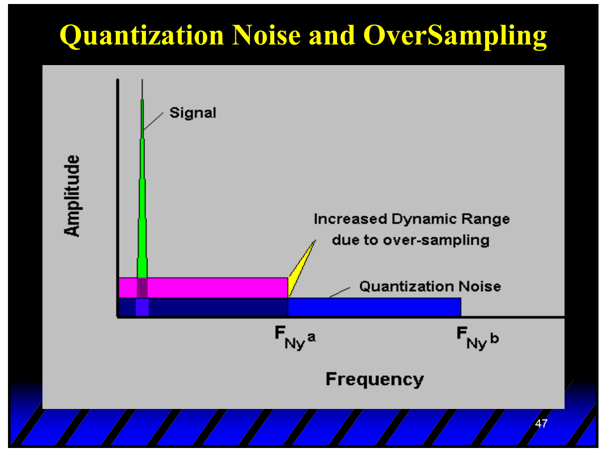


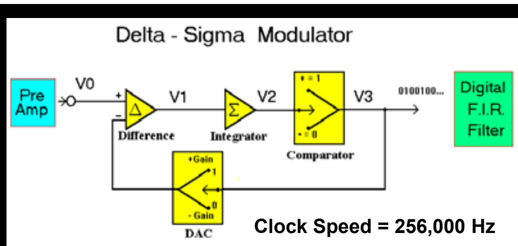
0.40 can be the average
of 10 binary bits
0 1 0 1 0
1 0 0 0 1
44



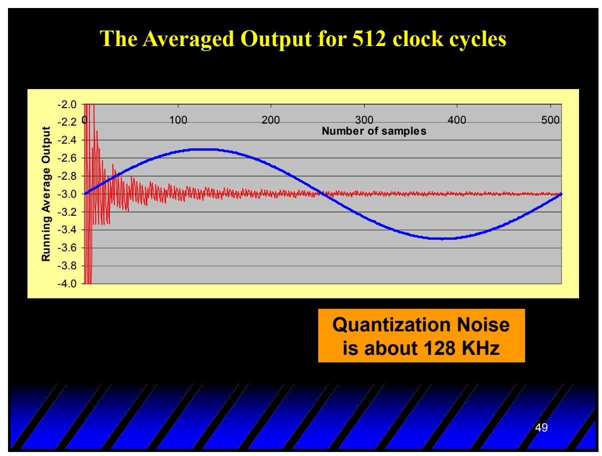
0.43 requires more precision and must
be estimated by the average
of 100 binary bits
0 1 0 1 0 0 1 0 0 1 0 1 1 0 1 0 0 1 0 1
1 0 0 1 0 0 1 0 1 0 0 1 0 1 0 0 1 0 0 1
1 0 0 1 0 1 0 1 0 1 1 0 0 1 1 0 0 1 0 1
0 1 0 0 0 0 1 0 0 1 0 1 0 0 1 0 0 1 0 1
1 0 0 1 0 1 0 0 0 1 1 0 1 0 1 0 0 1 0 1
45

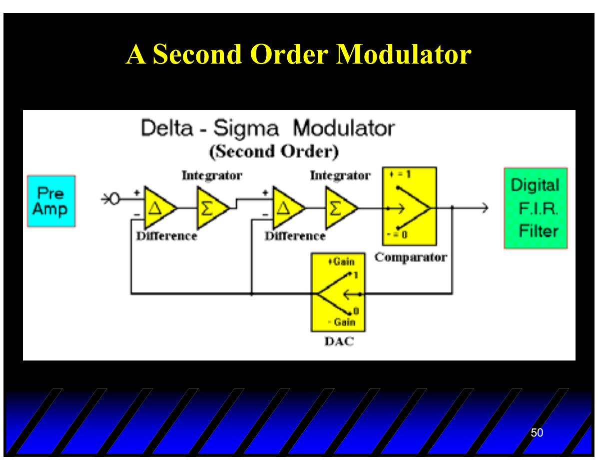


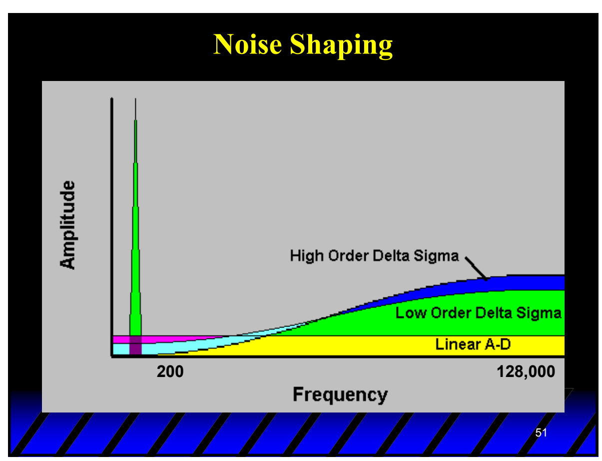


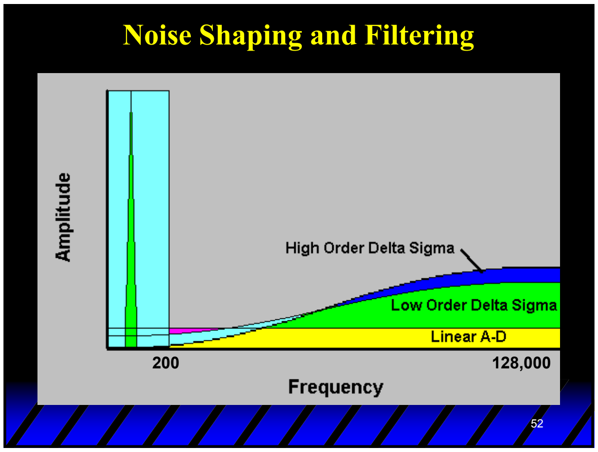


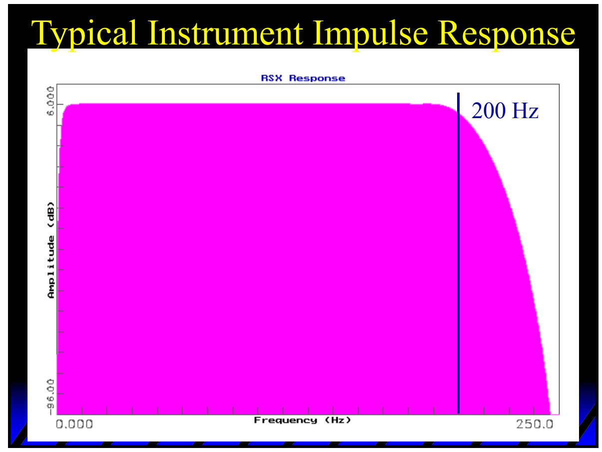
Cycle Number	Input Voltage	Difference Voltage	Integrated Voltage	1-Bit A/D	Running Average
0	0.00000	0.00000	0.00000	0	0.00000
1	-3.00000	-3.00000	-3.00000	-10	-10.00000
2	-3.00000	7.00000	4.00000	10	0.00000
3	-3.00000	-13.00000	-9.00000	-10	-3.33333
4	-3.00000	7.00000	-2.00000	-10	-5.00000
5	-3.00000	7.00000	5.00000	10	-2.00000
6	-3.00000	-13.00000	-8.00000	-10	-3.33333
7	-3.00000	7.00000	-1.00000	-10	-4.28571

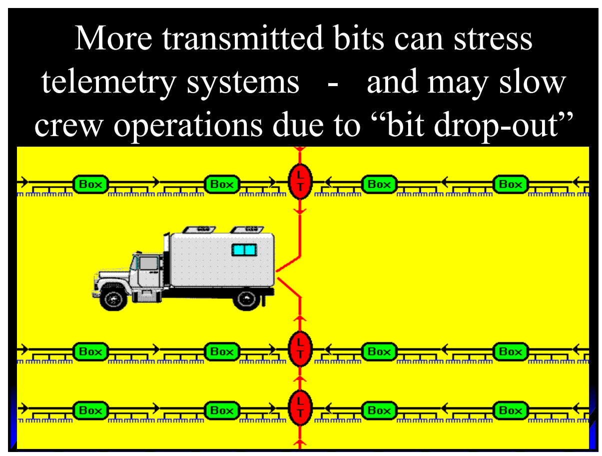












Conclusions

- Modern instruments faithfully record data up to the high cut filter (164 – 205 Hz depending on instrumentation)
- Loading up recording systems with unnecessary samples may cause telemetry problems and slow operations
- Oversampling refers to number of modulator samples per output sample. Decreasing this by decreasing output sample rate may diminish dynamic range

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Conclusions

- Data up to the Nyquist Frequency can be fully restored and downsampled by convolution with a finely sampled SYNC function.
- Processing algorithms such as NMO and sub-sample statics should use a SYNC interpolator when re-sampling data
- Finer sample intervals are not required until interpretation (for display of detailed data)

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