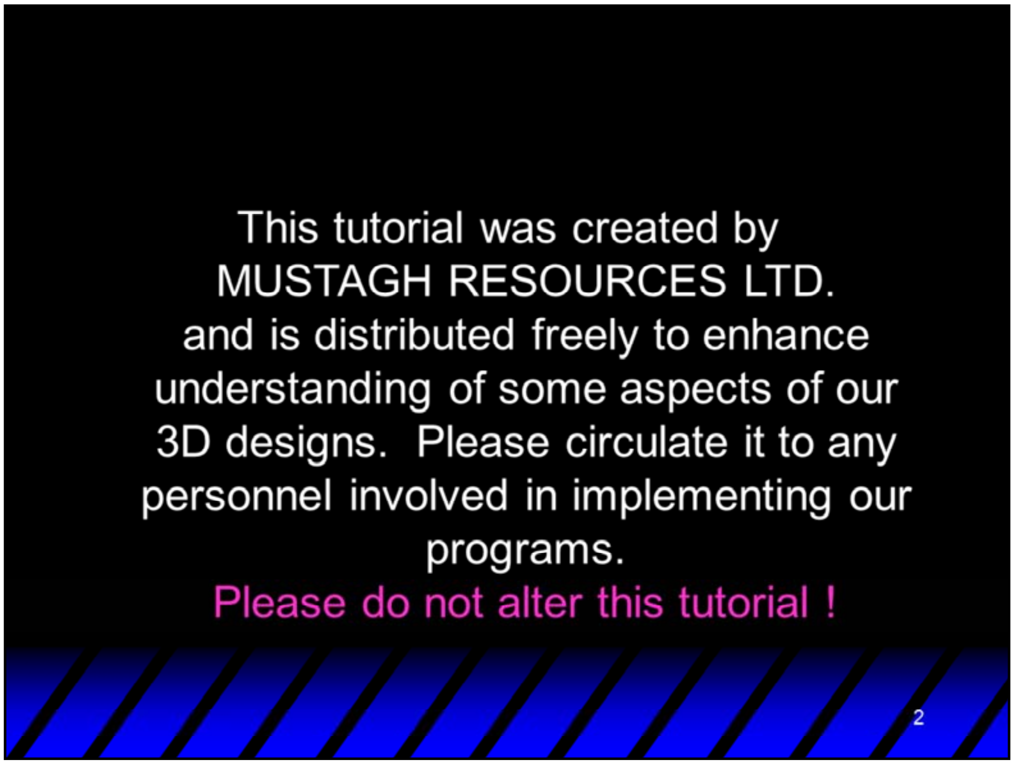
This presentation was prepared to illustrate the concept of "Effective" Length when discrete measurements are used to describe a changing behavior.

In particular, this has applications in geophysics where we measure amplitudes of geophone outputs in time as a discrete time series with a specified sample interval (often 0.002 seconds).

We also measure geophone outputs at discrete locations with a specified geophone or group interval (spatial sample interval).



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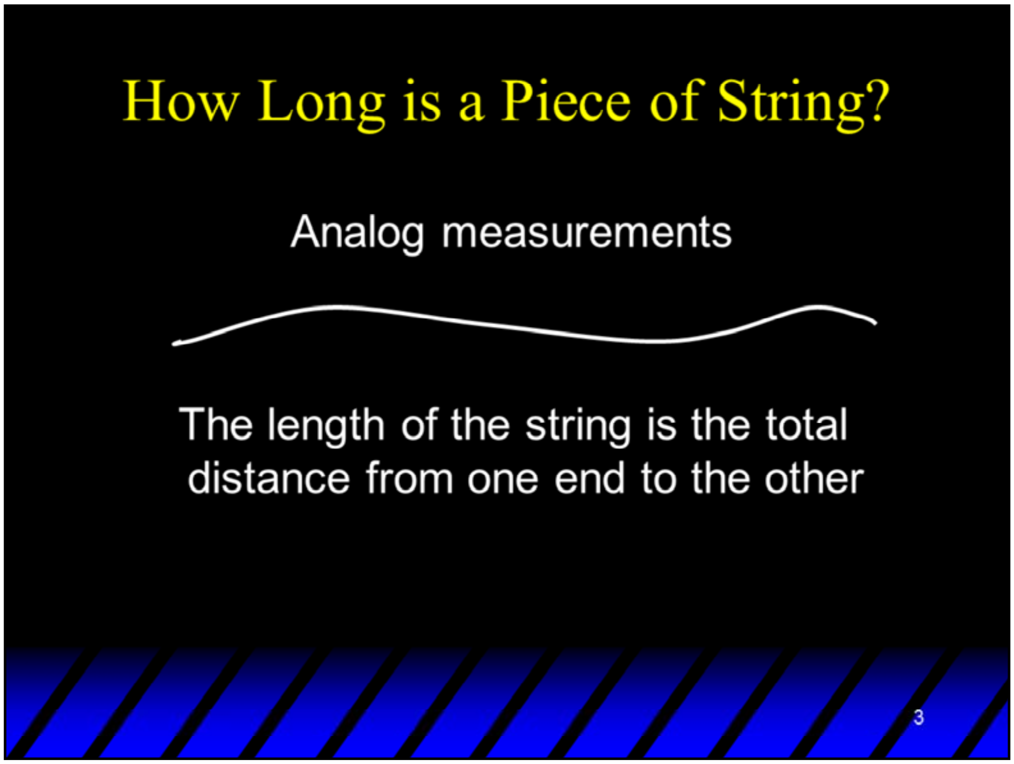
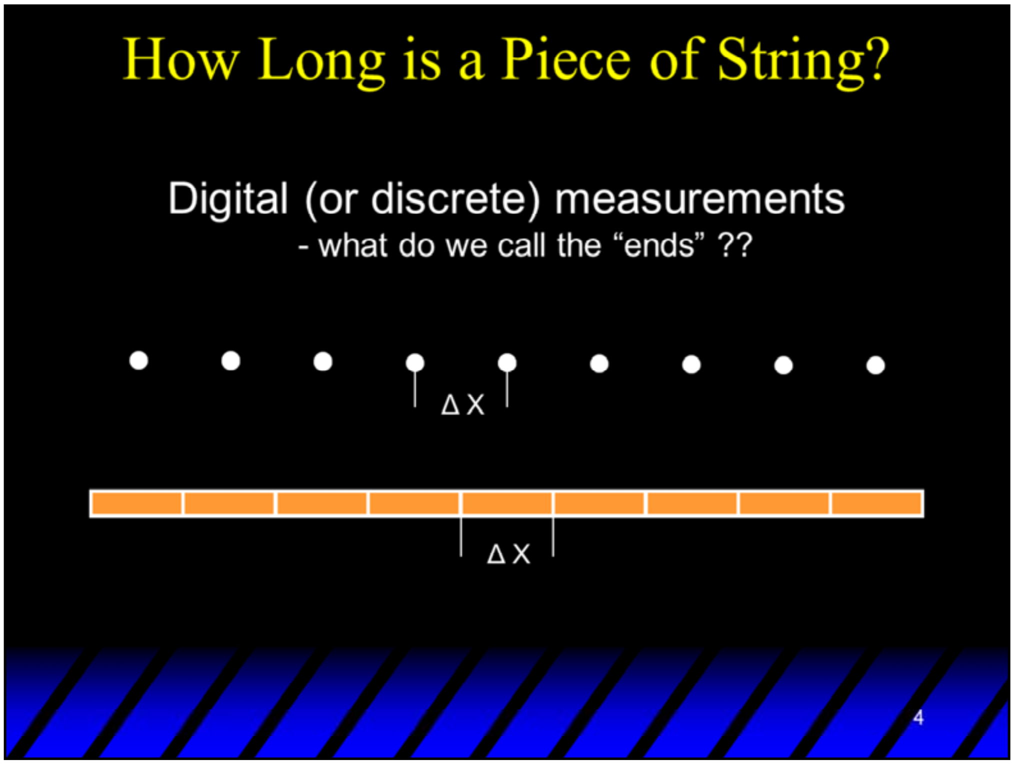


"A piece of string is twice as long as from the middle to one end."

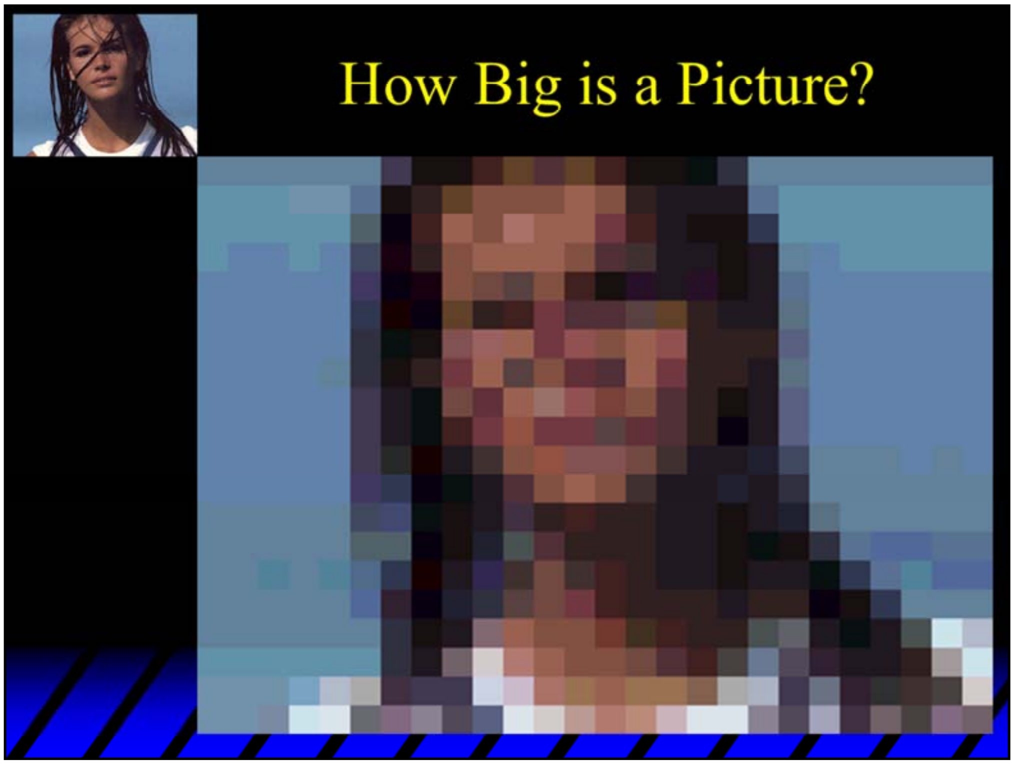
We use this simple question during some parts of our courses. But the simple answer is normally the distance along the string from one end to the other. For a continuous piece of string, it is generally clear what we mean by the "ends" of the string.

But what if the string is not continuous. What if it is a series of string segments?



9 points separated by a distance of ΔX will represent a distance from the first point to the last point of (9-1) x ΔX. This is called the "Measured" length and is given as (N-1) x S where N is the number of elements and S is the separation between elements.

However, if each point represents the center of a block of length ΔX, and 9 such blocks are laid end to end, then the length covered by these 9 blocks would be 9 x ΔX. This is called the "Effective" length and is given as N x S . It is important to understand when we should apply the measured length and when we should apply the effective length. Very often in geophysics, we are interested in the effective length. This is because usually we are using one discrete sample to represent what is occurring over one complete sample interval. For example, when we measure trace data at a temporal sample interval of 2 milliseconds, how many samples are necessary to define a 2-second trace. Some processing software will use one sample to define the amplitude of our trace at time 0.000, the next sample defines time 0.002, then 0.004 and so on until the last sample defines the amplitude of the trace at time 2.000 seconds. This would result in 1001 samples. However, if each digital time sample describes the average behavior of the trace during a 2 ms period, then a 2-second trace should contain 1000 samples. Some processing software uses the former convention and some uses the latter. However, the former is implying that the first and last sample are only representing a 1 millisecond sample of data, while all other samples are representing a 2 millisecond interval.

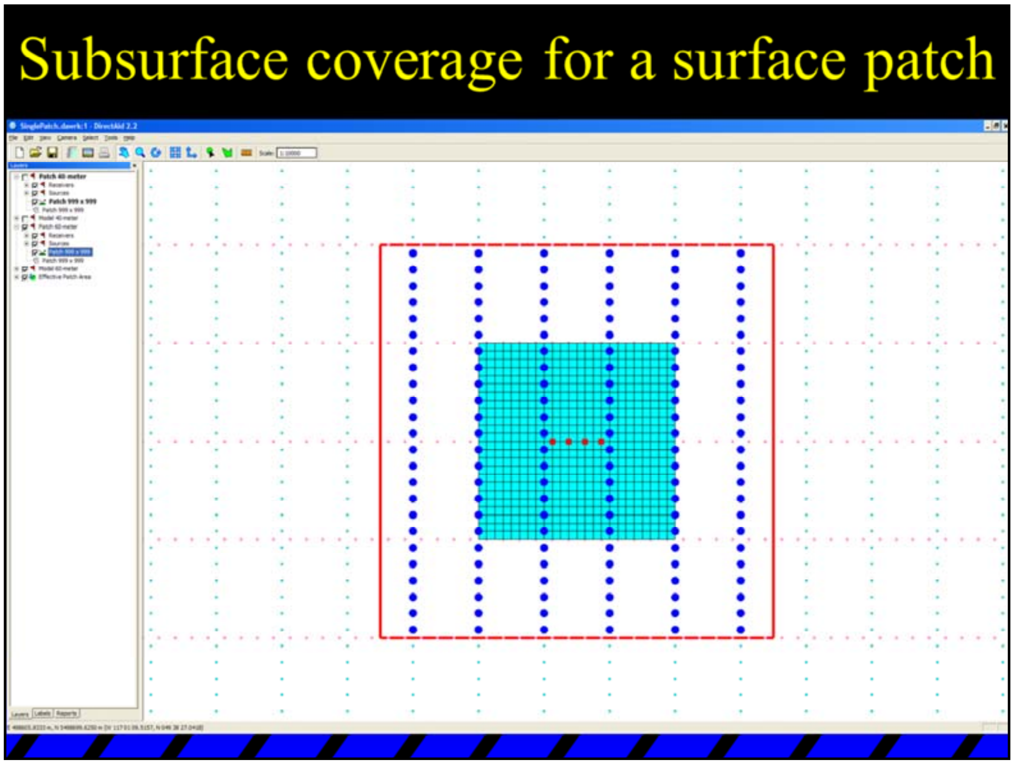


In the old days of cameras that recorded on film, a picture could be thought of as analog (although the grain density of the film actually meant that photos were discretely sampled). Now with digital photography, each pixel gives us an average color over that pixel size. The assemblage of pixels makes a photograph. Think of a jig-saw puzzle. On the cover of the box, we see a photo of the model Elle McPherson. However, we bought a 520-piece puzzle and each piece just has one color. The resulting picture has 26 x 20 pixels. If each piece of the jigsaw puzzle is 1 cm, then the final size of the assembled puzzle will be 26 cm x 20 cm (as per effective length) and NOT 25 cm x 19 cm (as per measured length).

If we use more pixels (more pieces in the jigsaw puzzle), we may see a better picture. In this case we have a puzzle with 2080 pieces. Each piece is 0.5 cm. We now see a picture with 52 x 40 pixels and its size is still 26 cm x 20 cm.

When we use discrete measured samples to represent the average value of some time or space interval, lengths are generally represented by the calculation called the “Effective” length.

Now let’s consider how this applies in geophysics.



Here is a 3D survey with an active patch of 6 lines by 24 stations (only for teaching purposes). When one salvo of source points is completed near the center of the patch, what subsurface coverage is obtained?

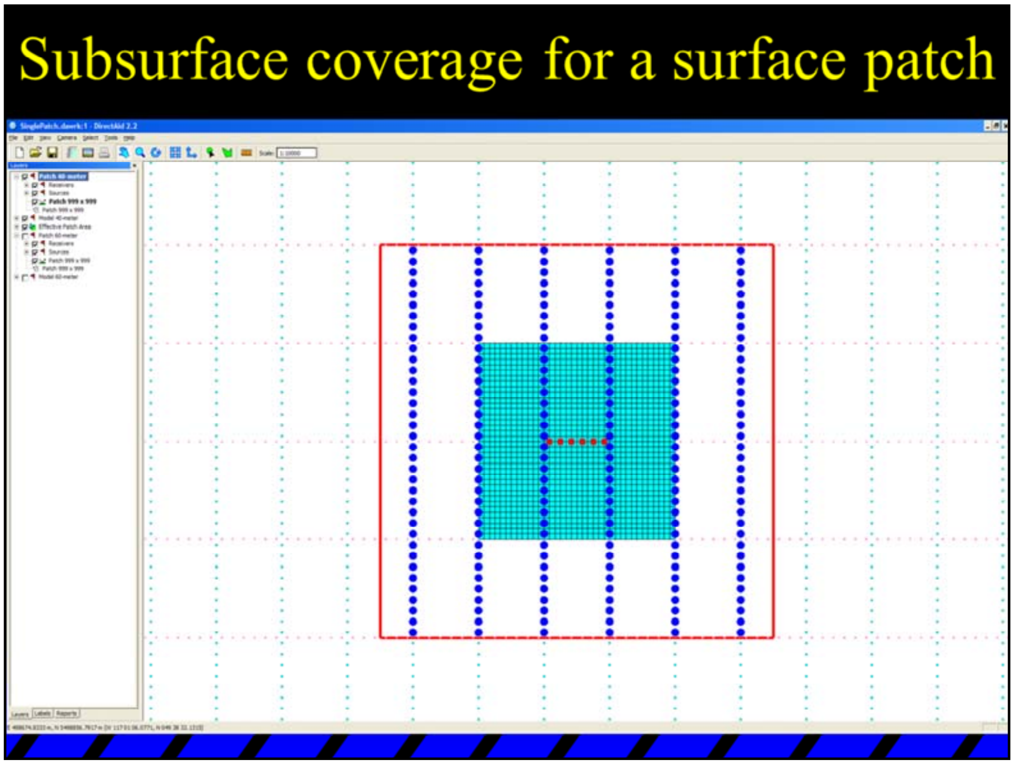
The effective in-line dimension of the surface patch is 24 stations x Ri and the effective cross-line dimension of the patch is 6 lines x RL. The subsurface imaged area is one half the effective cross-line width of the surface patch by one half the effective in-line height of the surface patch.

The spacing of additional source salvos is SL x RL.

This leads to the familiar 3D fold equation: Fold = Surface area of patch / (4 x SL x RL)

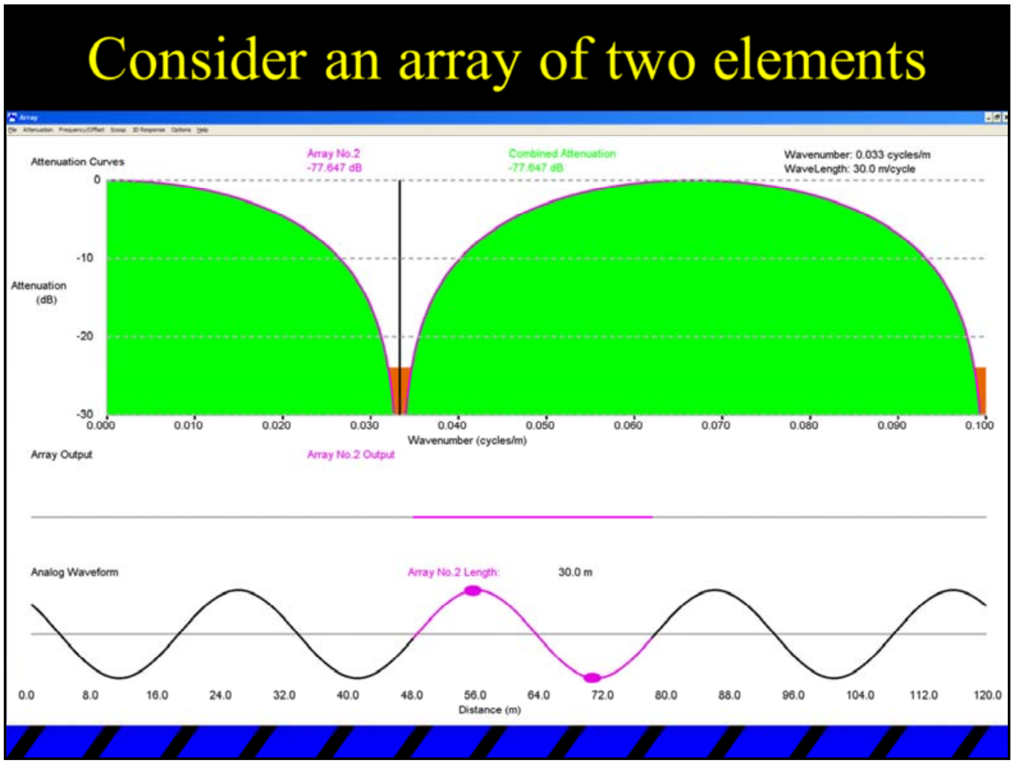
Again, Surface area of the patch is the product of the effective in-line dimension and the effective cross-line dimension (shown here by the red rectangle). For offset-limited fold calculations the effective surface area of the patch is simply □ $Xmax^2$ where Xmax is the maximum useful offset for the zone of interest. The effective width of the recording patch is what must be used for these equations to yield correct answers.

Note that each bin is represented by only one midpoint, but the full bin is considered to be imaged.



In this 3D survey, we have the same line spacings as in the previous example. However, we have now reduced the source and receiver interval. Even though the source interval is smaller, the subsurface imaged area remains exactly three receiver line spacings (one half the effective width of the surface patch. It is fundamental in a 3D that the subsurface imaged area is determined as ¼ the EFFECTIVE surface area of the recording patch. The natural sub-surface bin size is determined by ½ Si x ½ Ri. The fold is determined by the box area (SL x RL).

Again, for purposes of fold calculations and other statistics, the patch width is defined as the number of lines in the recording patch times the spacing of the receiver lines (i.e. "effective" width). Source interval has no impact on effective surface patch area.

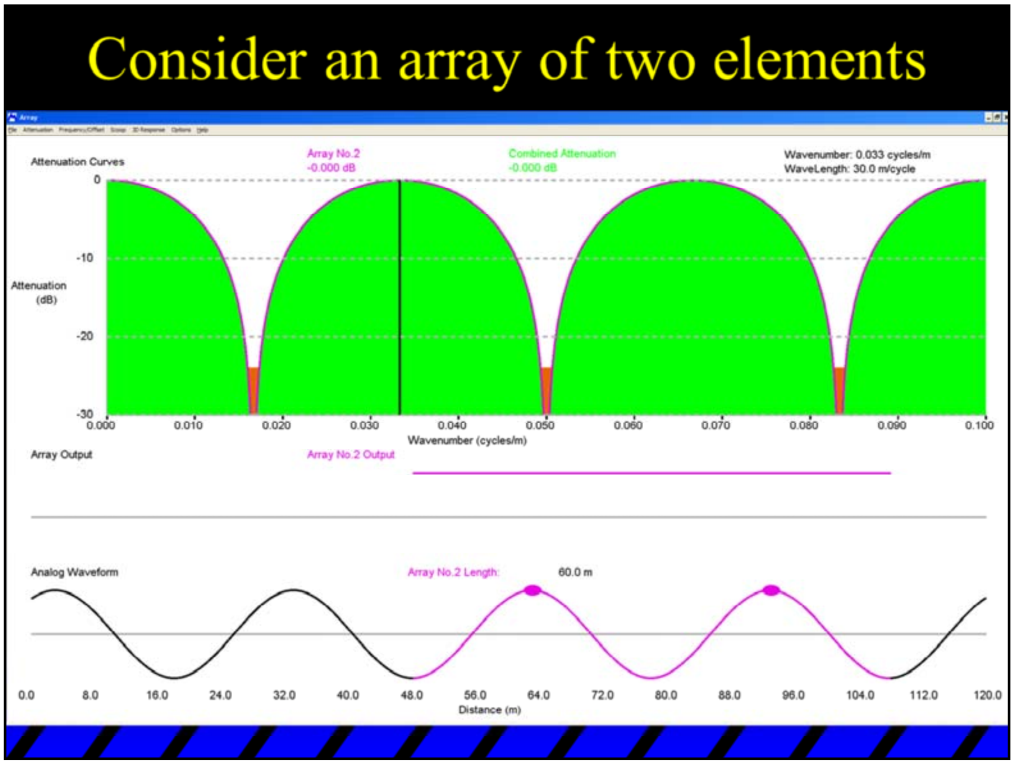


We sometimes use 2-element arrays. For example, a two-hole shot pattern; or using two vibrators per source point to produce energy. Imagine undesirable ground motion with an apparent wavelength of 30 meters (in our courses, we demonstrate that reflected signals of interest in oil and gas exploration generally have apparent wavelengths longer than 60 meters or wavenumbers less than 1/60 = 0.1667 m^{-1}). A two element array can theoretically be used to suppress certain wavelengths, although it has a very poor reject band.

The graph above shows that two elements spaced 15 meters apart would be separated by exactly half a 30-meter wavelength. Therefore, these two elements will always be 180 degrees different in phase for that wavelength and the output of the array will be very weak in the presence of 30 meter apparent wavelengths. The "Effective" length of the array is 2 x 15 = 30 meters. The array attenuates 30 meter wavelengths. The first "notch" in the response function is at a wavelength of 30 meters or a wavenumber of 1/30 = 0.0333 m^{-1}.

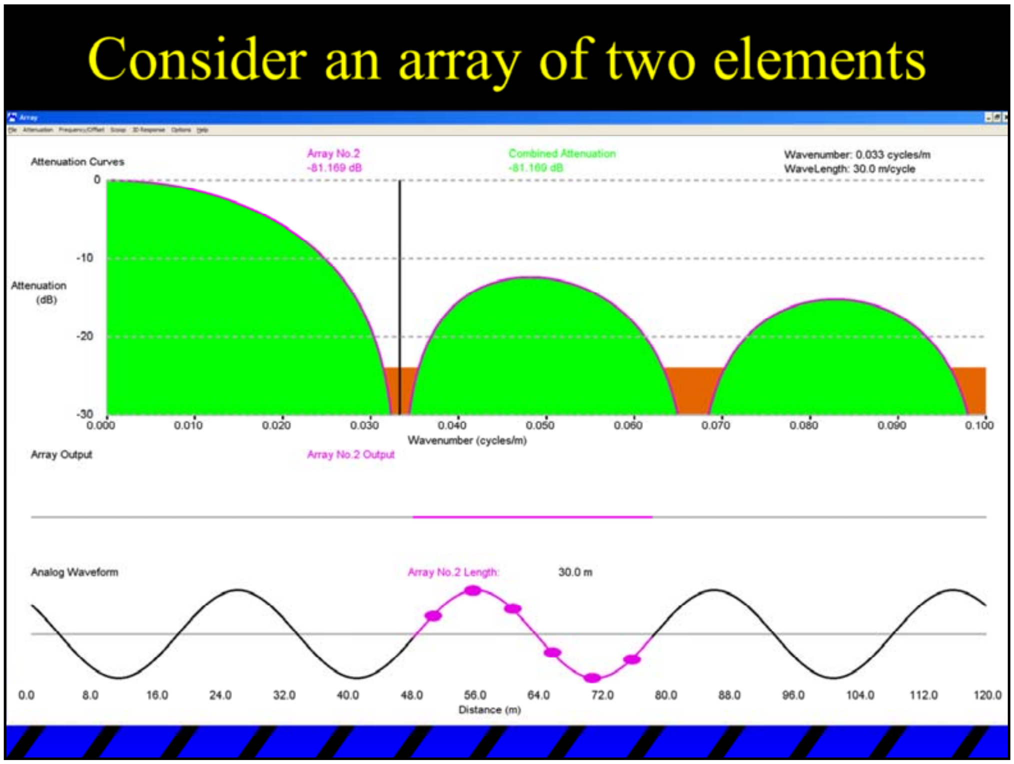
Of course, Newman and Mahoney pointed out in their 1973 paper "Patterns with a Pinch of Salt" that with realistic error in implementation (20%), then the best suppression we should expect in the "notches" is about -24 dB.

We fill the notches of our response curves with an orange color below -24 dB to acknowledge that the filter does not reach below this level.



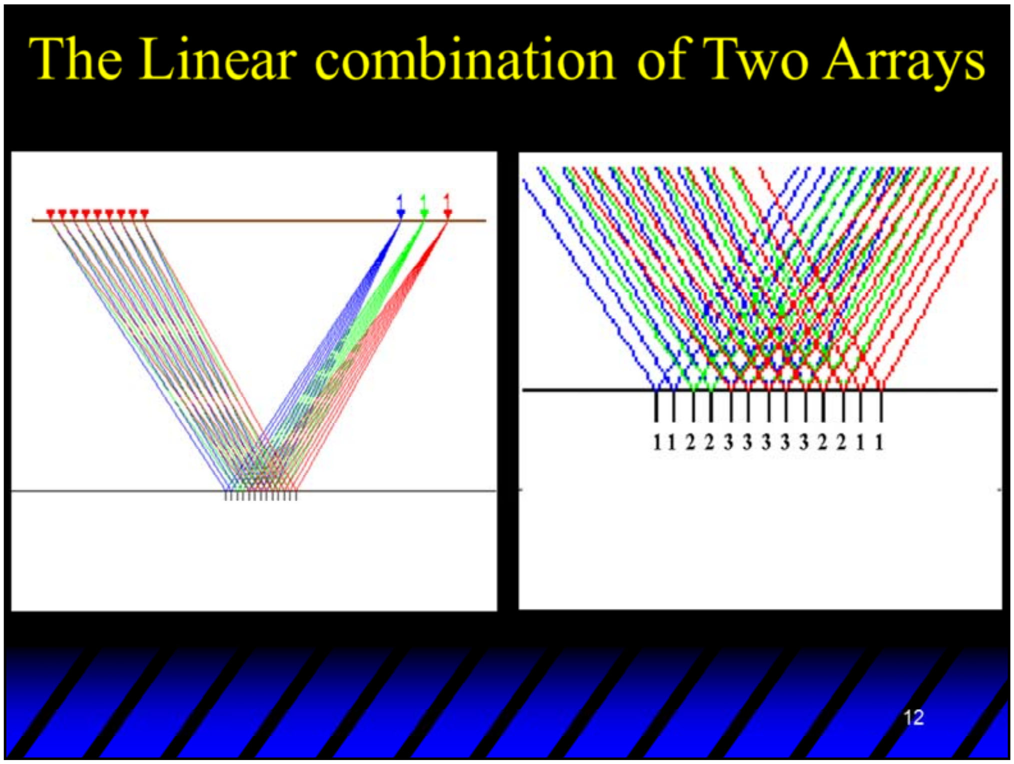
Some papers quote the "Measured" lengths of arrays and predict required spacing from that value. Note that if we use an element spacing of 30 meters we get a measured length of (2-1) x 30 = 30 meters. However, the effective length is 2 x 30 = 60 meters. Two elements separated by 30 meters will always have one cycle of a 30-meter apparent wavelength between elements. Both elements will always be measuring the same phase of the 30-meter waveform. Their outputs will not cancel each other as we would hope for a filter.

Notice that when measured length is used to calculate element spacing, the true notch of the array moves towards smaller wavenumbers (longer wavelengths) and may endanger desired signals. In this case the "notch" of the array filter coincides with wavelengths near 60 meters and that may include our shallow, high-frequency, far-offset reflections. We do not want to filter desired signals!



This is the response function for 6 geophones separated by 5 meters for an effective length of 30 meters. Again, notice that the effective length of the array coincides with the wavelength of the first notch in the response function. The second notch occurs when exactly two wavelengths exist across the array or at a wavenumber of 2/Le where Le is effective length (= N x ΔX) and ΔX is the spacing between individual geophones. We will have exactly two geophones per wavelength for a wavelength of 2 x ΔX (or a wavenumber of 1/(2 ΔX). This is the familiar Nyquist limit of discrete sampling and for this example the Nyquist wavenumber is K_{NY} = 0.100 m^{-1}. For an N-element array, the Nyquist limit is when N/2 wavelengths exist across the EFFECTIVE length of the array. The response for shorter wavelengths will be a mirror image of that shown above. The wavenumber that corresponds to the measured length of the array is 1/[(N-1) x ΔX] = 1/[(6-1) x 5] = .040 m^{-1}, which has no particular significance in the response function.

Again, the geophysically significant value comes from Effective length, not from Measured length.

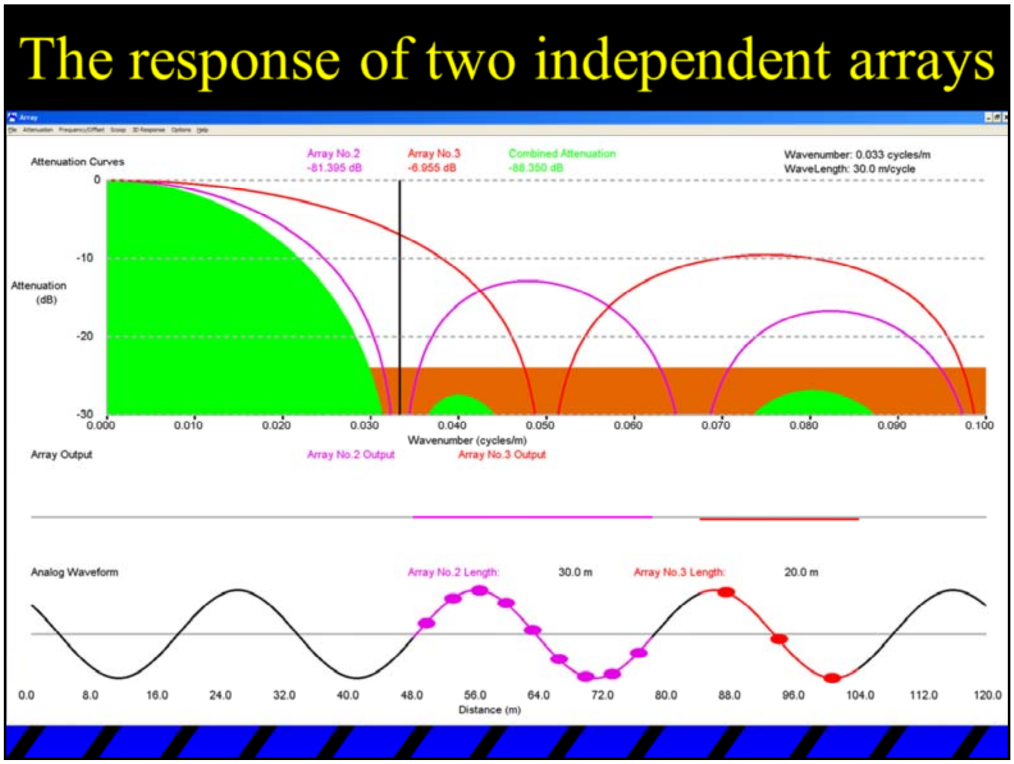


Consider a combination of a 9-element receiver array with an element spacing of ΔX and a 3-hole shot array with and hole spacing of $2\ \Delta X$. For 2-D programs the source array and receiver array will be in the same azimuth.

In terms of spatial functions sampled at ΔX , the geophone array represents the function 1-1-1-1-1-1-1-1-1 and the source array represents the function 1-0-1-0-1

The spatial convolution of these two functions is 1-1-2-2-3-3-3-3-3-2-2-1-1 as shown above. Independent arrays convolve in the spatial domain. That means that their response functions multiply in the wavenumber domain.
And if the response functions are expressed logarithmically (i.e. in deciBels), then the graphs simply add together.

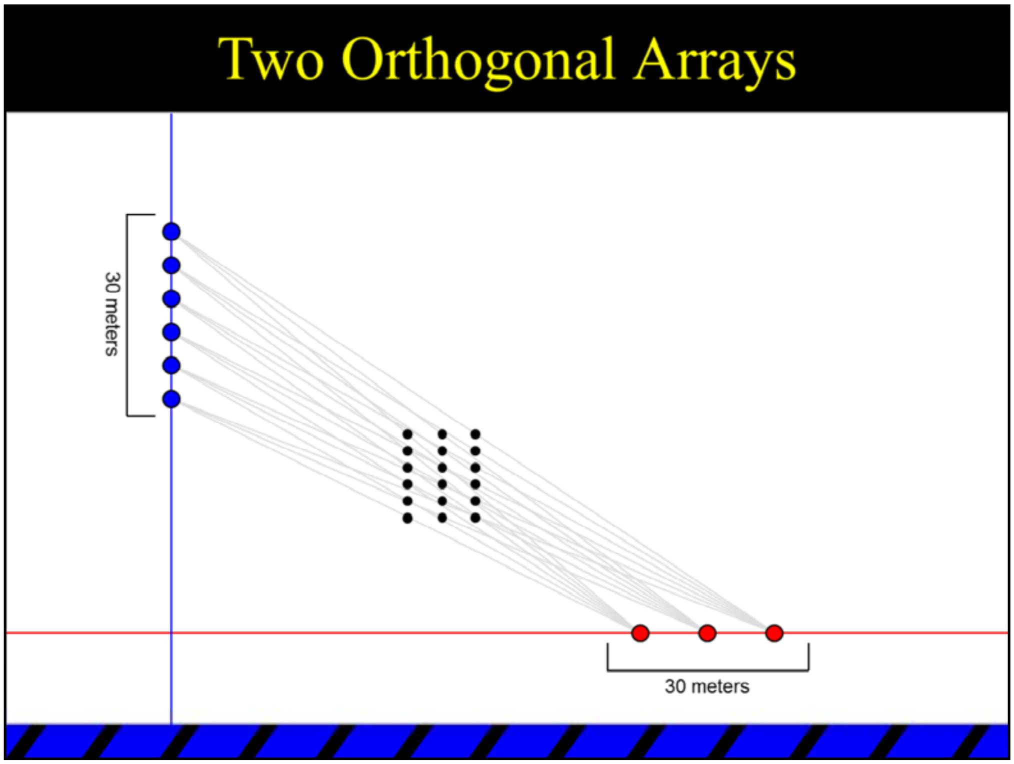
Notice that the Measured length of the receiver array is $8\ \Delta X$, the source array is $4\ \Delta X$ and the combined array is $12\ \Delta X$. Does this mean that the combined array is effectively longer? Will the combined array filter infringe on signal wavelengths?



A 9-element geophone array with ΔX=3.33333 meters will have an effective length of 30 meters. Its response is shown above in the violet line curve.

A 3-element source array with a hole separation of ΔX=6.66667 meters will have an effective length of 20 meters. Its response is shown above in the red line curve.

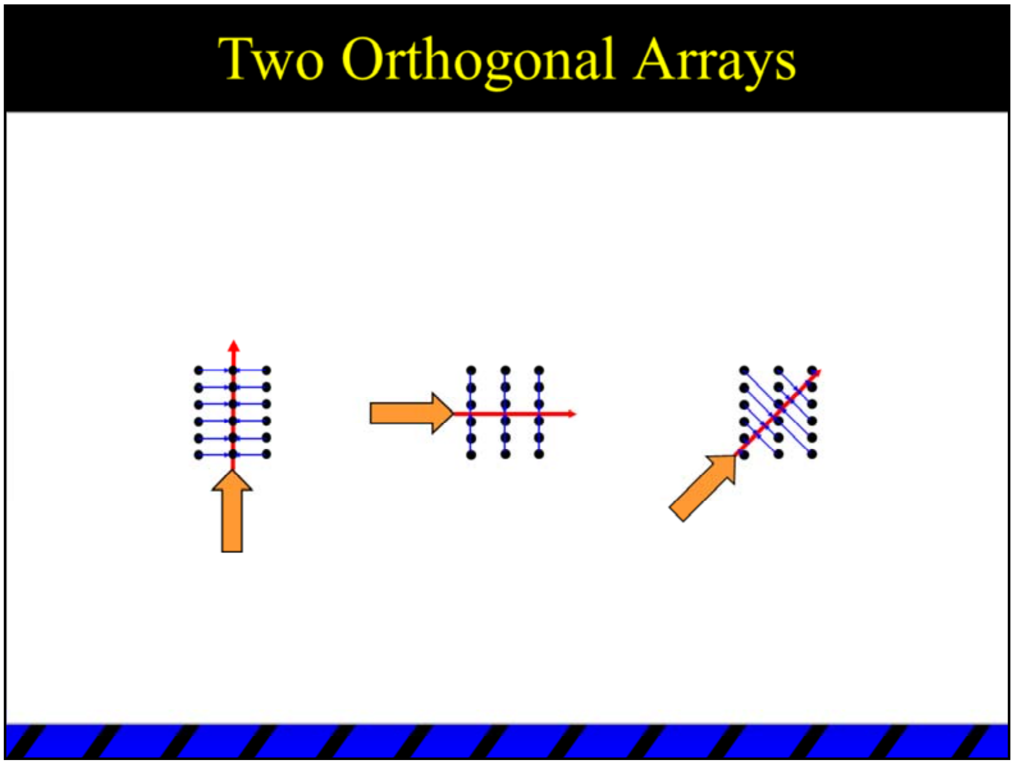
The combined response is indicated by the solid green curve. Note that the wavelength at the first notch coincides with the EFFECTIVE length of the longest contributing array (the geophone array). The notch for the shorter array moves to higher wavenumbers (shorter wavelengths) and simply adds more attenuation where the filter of the longer array was deficient. The EFFECTIVE length of a combination of independent arrays is equal to the longest contributing sub-array. Effectively, an array is not lengthened when it is used in combination with another array of equal or shorter effective length.



This diagram shows 6 geophones in-line along a north-south receiver line. The geophones are spaced 5 meters apart for an effective array length of 30 meters.

There are also 3 sources distributed along an east-west source line spaced 10 meters apart for an effective array length of 30 meters.

Notice that the combination of both arrays provides a 3x6 sub-sampling of the sub-surface midpoints spread over a sub-surface effective area of 15 x 15 meters (but the surface effective array is still 30 x 30 meters).
This is a rectangular function in space in two dimensions. … But what is the wavenumber transform of such a function?

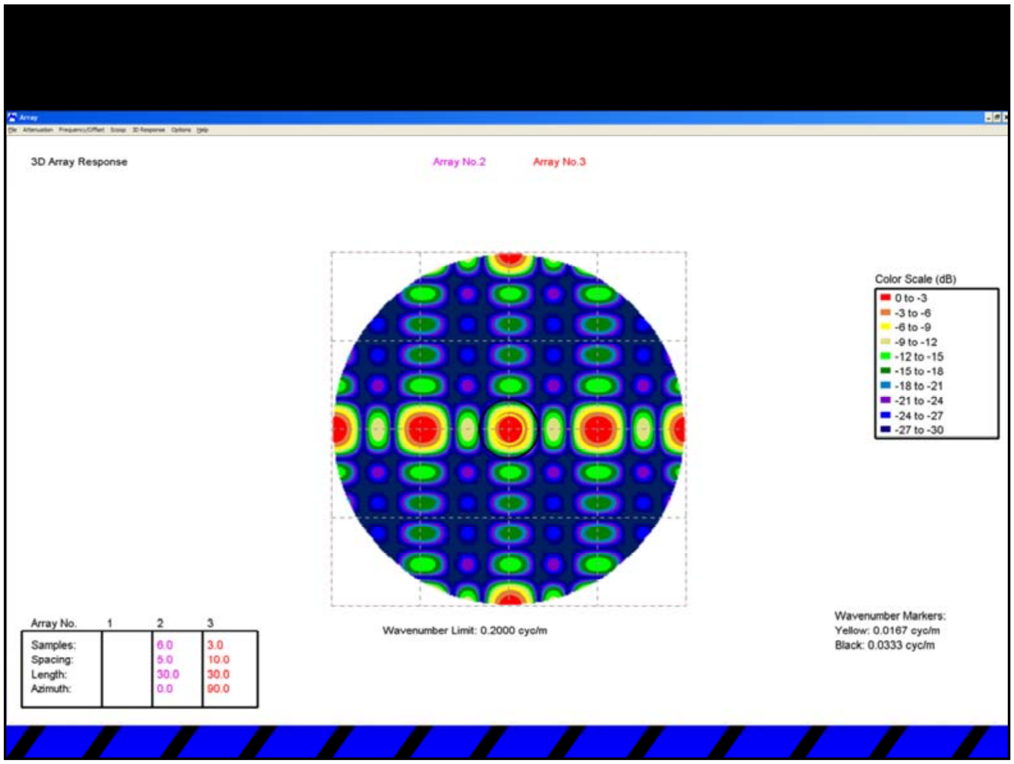


At the left, for waves traveling from south to north (S-R azimuth = 0), this array will behave like a linearly weighted array 3-3-3-3-3-3 which is simply a scaled version of a 6-element array with spacing of ΔX and effective length of 6 ΔX.

At the center, for waves traveling from west to east (S-R azimuth = 90), this array will behave like a linearly weighted array -6---6---6- which is simply a scaled version of a 3-element array with spacing of 2 ΔX and effective length of 6 ΔX.

However, at the right (S-R azimuth = 45), for waves traveling from southwest to northeast, this array will behave like a complex weighted array 1-1-2-2-3-3-2-2-1-1 but will still have an effective length of 6 ΔX.

For other source-receiver azimuths, the weighting function may be more complex and non-uniformly spaced. But its effective length will remain 6 ΔX.



The 3-D array response function shows the theoretical attenuation in dB as a color, wavenumber as radial distance from the center of the plot to the edge of the plot (wavenumbers of 0 to 0.2 m^{-1} or wavelengths of ∞ at the center down to 5 meters at the edge of the response. Source-receiver azimuth is represented by degrees of rotation around the plot.

All wavelengths outside the black circle are shorter than 30 meters and would be due to noise. All wavelengths inside the smaller yellow circle are longer than 60 meters and may contain signals of interest.

Notice that the central pass-band forms a circular response provided the source and receiver arrays have equal effective lengths.

Summary

We hope this presentation explains the importance of the concept of "Effective" distances.

When we discretely sample a continuous function at a uniform sample interval of ΔS, we expect that each sample summarizes the behavior of the function over a duration equal to the sample interval.

Therefore the portion of the function being measured is $N \Delta S$ where N is the number of samples measured.

Usually, "Effective" length is a more useful concept in digital theory than "Measured" length.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If you desire more information or would like a copy of this tutorial, please contact Norm Cooper or Yajaira Herrera
phone (403) 265-5255
fax (403) 609-3877
e:mail ncooper@mustagh.com
yajaira@mustagh.com
web page http://www.mustagh.com
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